MTHT 530 Analysis for Teachers II

Problem Set 7

1) Let $f(x) = 2x + 1$ on [1,3]. Let $P = \{1, \frac{3}{2}, 2, 3\}$.
a) Compute $L(f, P)$, $U(f, P)$ and $U(f, P) - L(f, P)$.

b) What happens if $Q = P \cup \{\frac{5}{2}\}$?

c) Find a partition P' where $U(f, P') - L(f, P') < 2$.

2) Prove that every increasing function $f : [a, b] \to \mathbb{R}$ is integrable. [Note for later: We really need only "nondecreasing", i.e., if $x < y$, then $f(x) \leq f(y)$.]

3) Suppose $f : [a, b] \to \mathbb{R}$ is integrable.
a) Show that changing the value of f at one point does not change the value of the integral.

b) Suppose $f : [a, b] \to \mathbb{R}$ is integrable and g differs from f at only finitely many points. Prove that g is integrable and $\int_a^b g = \int_a^b f$.

Bonus (5pt) a) Find an example showing that if we alter an integrable example at countably many points the new function might not be integrable.

b) Give an example of a function discontinuous at countably many points that is integrable. [Hint: use problem 2)]