MTHT 530 Analysis for Teachers II
Problem Set 7

Due: Wednesday March 15

1) Let $f : [0, 1] \to \mathbb{R}$ be the function

$$f(x) = \begin{cases} 1 & \text{if } x = \frac{1}{n} \text{ for } n = 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}.$$

We will prove that f is integrable.

a) Prove that $L(f, P) = 0$ for all partitions.

b) Suppose $\epsilon > 0$. Find a partition P such that $U(f, P) < \epsilon$.

c) Conclude that f is integrable and $\int_0^1 f = 0$.

2) Prove or give a counterexample.

a) If $|f|$ is integrable on $[a, b]$, then so is f.

b) Assume g is integrable and $g \geq 0$ on $[a, b]$. If $g(x) > 0$ for an infinite number of points $x \in [a, b]$ then $\int_a^b g > 0$.

c) If g is continuous on $[a, b]$ and $g \geq 0$ with $g(x_0) > 0$ for at leat one point in $[a, b]$, then $\int_a^b g > 0$.

d) If $\int_a^b f > 0$, there is an interval $[c, d] \subseteq [a, b]$ and $\delta > 0$ such that $f(x) \geq \delta$ for all $x \in [c, d]$.

3) Suppose f is uniformly continuous on $(a, b]$ and on $[b, c)$. Prove that f is uniformly continous on (a, c).