MTHT 530 Analysis for Teachers II
Midterm II Study Guide

The webpage

`http://www.math.uic.edu/~marker/mtht530/concepts.html`

contains a summary of all key concepts and results we have discussed in the course.

Sample Exam Questions

1) Let $f : [a, b] \to \mathbb{R}$ be a bouned function. Give a complete definition of what it means for f to be integrable and $\int_a^b f = L$, (including the definition of upper and lower sums).

2) State both versions of the Fundamental Theorem of Calculus.

3) Decide if the following statements are TRUE or FALSE. If FALSE, prove justify your answer.

a) If $f : [a, b] \to \mathbb{R}$ is differentiable, then f' is integrable.

b) Every differentiable $f : [a, b] \to \mathbb{R}$ is integrable.

c) If f is one-to-one and differentiable at $a = f(b)$, then f^{-1} is differentiable at b.

d) If $f : [a, b] \to \mathbb{R}$ is integrable, $f(x) \geq x$ for all x and $f(c) > 0$ for some $a < c < b$, then $\int_a^b f > 0$.

4) Use the tangent approximation to approximate $\sqrt{25.5}$.

5) Find $(f^-1)'(0)$ where

$$f(x) = \int_0^x 1 + \sin(\sin t)\, dt$$

6) Prove that if $f(x) \leq g(x)$ for all $x \in [a, b]$ and f, g are integrable, then $\int_a^b f \leq \int_a^b g$.

7) Suppose $f : [a, b] \to \mathbb{R}$ is integrable. Prove that there is $a \leq c \leq b$ such that $\int_a^c f = \int_c^b f$.

8) Let $F(x) = \int_2^x \frac{1}{\ln t}\, dt$. Prove that F is not bounded on $[2, +\infty)$.