Exercises on Büchi automata:

Exercise 1 Let $\mathfrak{A} = (S, I, T, F)$ be the Büchi automaton defined by $S = \{s_0, s_1, s_2\}$, $I = \{s_0\}$, $\Sigma = \{a, b\}$, $T = \{(s_0, a, s_0), (s_0, b, s_1), (s_1, b, s_1), (s_1, b, s_2), (s_2, a, s_2)\}$, $F = \{s_2\}$.

(a) How many runs does $\mathfrak{A}$ have on the input aaabbbaaaaa.... ?

(b) Find an input on which $\mathfrak{A}$ has no runs.

(c) Is there an input on which $\mathfrak{A}$ has infinitely many runs ?

(d) Find $L(\mathfrak{A})$.

Exercise 2 Let $\mathfrak{A} = (S, I, T, F)$ be the Büchi automaton on the alphabet $\Sigma = \{a, b\}$ defined by

$S = \{s_0, s_1, s_2\}$, $I = \{s_0\}$, $F = \{s_2\}$,

$T = \{(s_0, a, s_0), (s_0, b, s_1), (s_1, a, s_2), (s_1, b, s_1), (s_2, b, s_1), (s_2, a, s_0)\}$.

(a) Draw a graph representing $\mathfrak{A}$.

(b) Find $L(\mathfrak{A})$.

Exercise 3 Let $\Sigma = \{a, b\}$. Construct Büchi automata recognizing the following languages

(a) $\{\alpha \in \Sigma^\omega \mid a \text{ occurs exactly once in } \alpha\}$

(b) $\{\alpha \in \Sigma^\omega \mid \alpha \text{ contains no substring } aa\}$

(c) $\{\alpha \in \Sigma^\omega \mid \alpha \text{ contains infinitely many } b\text{'s}\}$

(d) $\{\alpha \in \Sigma^\omega \mid$ after each a occurring in α there is either an even number of consecutively occurring b's or an infinite number $\}$

(e) $\{\alpha \in \Sigma^\omega \mid \alpha = (ab)^n \beta, \ n \geq 1, \ \beta \in \Sigma^\omega\}$.

(f) $\{\alpha \in \Sigma^\omega \mid a \text{ occurs infinitely often in even positions}\}$.

Exercise 4 Let $V, W \subseteq \Sigma^*$ and fix $m \geq 1$

Let $L \subseteq \Sigma^\omega$ be the language consisting of all

$$\alpha = x_1 x_2 x_3 x_4 \cdots$$

where $x_i \in V \cup W$ and

(i) if $x_i \in W \setminus V$, then we do not have

$$x_i = x_{i+1} = \cdots = x_{i+m}$$

(ii) if $x_i \in V \setminus W$, then we do not have

$$x_i = x_{i+1} = \cdots = x_{i+m} .$$

Show that if V, W are regular, then L is Büchi recognizable.