# FORCING EXERCISES
# DAY 3

Problem 1. Give an example of a poset for which the antichains are exactly the almost disjoint families of subsets of $\omega$.

Problem 2. Recall the definition of *open* in a poset. Show that $\mathrm{MA}(\kappa)$ is equivalent to the same statement where the word 'dense' is replaced with the words 'dense open'.

Problem 3. An antichain is *maximal* if it is maximal under containment. Show that $\mathrm{MA}(\kappa)$ is equivalent to the formulation where 'dense sets' is replaced with 'maximal antichains'.

Definition 1. Let $\mathbb{P}$ be a poset.

1. $\mathbb{P}$ is **$\sigma$-centered** if there is a partition of  into set $P_i$ for $i < \omega$ such the for all $i$ and all $p, q \in P_i$, $p$ and $q$ are compatible.
2. $\mathbb{P}$ is **$\aleph_1$-Knaster** if for every sequence $\langle p_\alpha \mid \alpha < \omega_1 \rangle$ of elements of $\mathbb{P}$, there is an unbounded $I \subseteq \omega_1$ such that for all $\alpha, \beta \in I$, $p_\alpha$ is compatible with $p_\beta$.

Problem 4. Show that for a poset $\mathbb{P}$, $\sigma$-centered implies $\aleph_1$-Knaster implies ccc.

Problem 5. Let $\mathbb{P}$ be as in the proof that MA implies $\mathfrak{b} = 2^{\aleph_0}$.

1. Give an explicit example of a maximal antichain in $\mathbb{P}$.
2. For each $k \in \omega$, show that $\{(p, A) \mid k \in \mathrm{range}(p)\}$ is not dense.

Problem 6. Prove Theorem 4.3 from the notes. (There are hints in the notes.)