FORCING EXERCISES
DAY 4

PROBLEM 1. Show that MA implies $\mathfrak{t} = 2^\omega$.

PROBLEM 2 (*). Show that MA implies $\mathfrak{s} = 2^\omega$.

PROBLEM 3. Suppose that $\{A_n \mid n < \omega\} \subseteq \mathcal{L}$ and for all n, $A_n \subseteq A_{n+1}$. Show that $\mu(\bigcup_{n<\omega} A_n) = \lim_{n<\omega} \mu(A_n)$.

PROBLEM 4. Show there is a dense G_δ (equivalently, $\mathbf{\Pi}^0_2$) set with Lebesgue measure zero.

PROBLEM 5. We define a poset $\mathbb{P}$. Conditions are subsets of the interval $(0, 1)$ which have positive Lebesgue measure and they are ordered by $p \leq q$ if and only if $p \subseteq q$.

1. Give a characterization of $p \perp q$ for $p, q \in \mathbb{P}$.
2. Show that if $\{p_n \mid n < \omega\}$ is an antichain, then $\mu(\bigcup_{n<\omega} p_n) = \sum_{n<\omega} \mu(p_n)$.
3. Show that $\mathbb{P}$ is ccc.

PROBLEM 6. Do the following.

1. Show that every open subset of $\mathbb{R}$ can be written as the union of open intervals with rational endpoints.
2. Show that there are exactly $2^{\aleph_0}$ open sets.

PROBLEM 7. Show that there are exactly $2^{\aleph_0}$ many Borel sets.

PROBLEM 8. Show that there are Lebesgue measure zero sets which are not Borel. (Hint: Think about the Cantor set.)