CS 501 / MCS 501 – Computer Algorithms I

Fall 2020

Problem Set 3

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Due: 10/19/20 by the beginning of class

Instructions: Atop your problem set, write your name and also write the names of all the students with whom you have collaborated on this problem set.

1. [**15 pts**] Solve the following problems related to Steiner trees (and prize-collecting Steiner trees).

> **a.** [**5 pts**] Exercise 2.5 (a) from Williamson and Shmoys.
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> **b.** [**5 pts**] Exercise 2.5 (b) from Williamson and Shmoys.
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> **c.** [**5 pts**] Use the problems above to design an algorithm (like algorithm 4.1 in Williamson and Shmoys but without employing Exercise 7.6) and use it to prove Lemma 4.6 in Williamson and Shmoys.

2. [**20 pts**] Consider the knapsack problem as described in Williamson and Shmoys. In this homework problem we will prove an approximation guarantee for a simple greedy algorithm. (This guarantee will not be as good as the FPTAS that is possible by using dynamic programming.)

> **a.** [**5 pts**] Consider the following greedy algorithm for the knapsack problem. Start by reordering the items by ratio of value to size so that $v_1/s_1 \geq v_2/s_2 \geq \ldots \geq v_n/s_n$. Now, fill the knapsack in the above order until the next item no longer fits. Give a bad example for this algorithm and conclude that no constant factor approximation guarantee for it is possible.
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> **b.** [**5 pts**] Consider the following greedy algorithm for the knapsack problem. Reorder the items according to their value (ignoring their sizes), so that $v_1 \geq v_2 \geq \ldots v_n$. Add the items to the knapsack in the above order until the next item no longer fits. Give a bad example for this algorithm and conclude that no constant factor approximation guarantee for it is possible.
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> **c.** [**10 pts**] Prove that if OPT is the value of the optimal solution, either the algorithm in a) produces a value that is $\geq$OPT/2 or that the most valuable item has value $\geq$OPT/2. Conclude by arguing that running the two greedy strategies in parallel and choosing the better of the two solutions gives a 1/2-approximation.

3. [**10 pts**] Use the method of conditional expectations to derandomize the 1/2-approximation of weighted max cut (where each vertex is placed equiprobably on either side of the cut).