STAT 473 – Game Theory
Fall 2021
Problem Set 1

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Due: 9/21/21, 9:30 am

1. [10 pts] Consider the following two-person zero-sum game. Two players simultaneously say one of the numbers 2 or 3. Plater 1 wins if the sum of the numbers is odd, and Player 2 wins if the sum of the numbers is even. The loser pays the winner (in dollars) the product of the two numbers that were said. Give the payoff matrix, the value of the game, and optimal strategies for both players.

2. [10 pts] Two players each choose a positive integer. The player who chose the lower number pays \$1 to the player who chose the higher number (and no money is exchanged if the two players choose the same number). Explain why this game has no Nash Equilibrium. Determine the safety values for the two players (and explain why the safety values are the ones that you gave).

3. [10 pts] Find a 2-player zero-sum game where player 1's unique optimal strategy is pure and player 2's optimal strategies are all mixed.

4. [20 pts] Find the values of the zero-sum gave given by the following payoff matrices and determine an optimal pair of strategies for both players. Explain how you arrived at your answer.

   i. [10 pts]
$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \\ 1 & 1 \end{pmatrix}$$

   ii. [10 pts]
$$\begin{pmatrix} -1 & 3 & 2 \\ 0 & 3 & -2 \\ 4 & 0 & 1 \end{pmatrix}$$