STAT 473 – Game Theory
Fall 2021
Problem Set 3

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Due: 11/9/21, 9:30 am

For problems **1** and **2** below, consider the modified game of Rock-Paper-Scissors in Figure 1 where players are both punished when they both play the same action.

	R	P	S
R	(-1,-1)	(-1,1)	(1,-1)
P	(1,-1)	(-1,-1)	(-1,1)
S	(-1,1)	(1,-1)	(-1,-1)

Table 1: The payoff matrix for players 1 and 2 of modified Rock-Paper-Scissors.

1. **[10 pts]** Find a Nash equilibrium of the game in Figure 1. What is the expected payoff to both players? Is this equilibrium evolutionarily stable? Why or why not?

2. **[10 pts]** Find a correlated equilibrium of the game in Figure 1 that results in an expected higher payoff (than the Nash equilibrium) to both players and explain why that distribution is in fact a correlated equilibrium.

3. **[10 pts]** Consider a game where two players simultaneously choose A or B and both get a payoff of 1 if they choose the same letter and both get a payoff of 0 if they choose different letters. Is both players playing $(1/2, 1/2)$ a Nash equilibrium? If so, is $(1/2, 1/2)$ also an evolutionarily stable equilibrium? Why or why not?

4. **[10 pts]** Consider a similar game where where two players simultaneously choose A or B and both get a payoff of 1 if they choose different letters and both get a payoff of 0 if they choose the same letter. Is both players playing $(1/2, 1/2)$ a Nash equilibrium? If so, is $(1/2, 1/2)$ also an evolutionarily stable equilibrium? Why or why not?

5. **[10 pts]** Consider the game "Golden Balls" where two players must divide a pot of money by each choosing *split* or *steal*. If both players choose *split*, the pot is divided evenly. If one chooses *split* and the other chooses *steal*, the player who chose *steal* gets the entire pot. If both choose *steal*, both get nothing. In class, we noted that *steal* is a dominant strategy and (*steal*,*steal*) is a Nash equilibrium. However, we might ask whether there exists a correlated equilibrium that gives a non-zero expected payoff to each player. Does such an equilibrium exist? If so, what is one such equilibrium and why? If not, why not?