MCS 401 – Computer Algorithms I
Spring 2016
Problem Set 2

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Due: 2/15/16 by the beginning of class

Instructions: Atop your problem set, write your name and whether you are an undergraduate or graduate student. Also write the names of all the students with whom you have collaborated on this problem set.

Important note: Problems labeled "(**U**)" and "(**G**)" are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice.

1. [10 pts] Consider a rooted, connected graph $G = (V, E)$, and a vertex $u \in V$. Let T_B be a BFS tree of G rooted at u, and let T_D be a DFS tree of G rooted at u. Prove that if $T_B = T_D$, then G is a tree.

2. [10 pts] It is well known that a priority queue can be used to implement a sorting algorithm. Describe the details of how to use a priority queue to sort a list of N integers. Assume you will use a priority queue with the following running times: $f(n)$ time to `ExtractMin`, $g(n)$ time `FindMin`, and $h(n)$ time `Insert`, where n is the number of elements in the queue when the respective operations are called. What is the resulting running time of your sorting algorithm as a function of N (and of f, g, and h)?

3. [10 pts]

(**U**) A large number of houses are being built on a straight street in a new development. As the city planner, you need to place fire hydrants such that each house is within 300 feet of one before anyone can move in. To save costs, you want to place as few fire hydrants as possible. Formalize this problem and devise an algorithm to optimally solve this problem. Prove your algorithm to be correct.

(**G**) In this Algorithms class, I want to cover topics $t_1, \ldots, t_n$. I should present all the n topics in that exact order because each topic t_i requires the students to have seen the material in topics t_1 to t_{i-1}. Each topic i takes no more than 50 minutes to present, and I have 50 minutes per lecture to cover as many topics as I'd like. I have also learned (the hard way) that I cannot split topics across lectures; otherwise students have a hard time following my lectures. Currently, I use the following simple strategy: on a given lecture, I cover as many topics as I can back-to-back, and I stop once the time remaining in the lecture is smaller than

the amount of time I need to cover the next topic. Then, I let students ask me questions (or otherwise use the remaining time) or simply end class a little early.

However, I recently had an idea. What if I sometimes didn't fill up a lecture with topics as fully as possible, so that some of the later lectures would be better "packed"? My hope is that if I change my strategy, we might be able cover all the material for this class in fewer lectures. Then I could finish teaching early and fly off to Montana to do some hiking in Glacier National Park, which I hear is beautiful in Spring. Could my idea possibly work, or are we all stuck here until the end of the semester? Formalize this problem and then prove why or why not.

4. [10 pts] Suppose we are given an instance of the Shortest s–t Path Problem on a directed graph G. We assume that all edge costs are positive and distinct. Let P be a minimum-cost s–t path for this instance. Now suppose we replace each edge cost c_e by its square, c_e^2, thereby creating a new instance of the problem with the same graph but with different costs. Prove or disprove the following claim: P must be a minimum-cost s–t path in this new instance.