MCS 401 – Computer Algorithms I
Spring 2016
Problem Set 4

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Due: 3/18/16 by the beginning of class

Instructions: Atop your problem set, write your name and whether you are an undergraduate or graduate student. Also write the names of all the students with whom you have collaborated on this problem set.

1. **[10 pts]** A rather famous sequence is defined as $F_0 = 1,\ F_1 = 1$, and for $n > 1$, $F_n = F_{n-1} + F_{n-2}$. A natural recursive algorithm to calculate the value of F_n is shown below, in Algorithm 1.

Algorithm 1 F(int n)

```
if (n = 0 or n = 1) then return 1;
else return F(n-1) + F(n-2)
end if
```

Does this algorithm compute F_n in time polynomial in n? Why or why not? Give a dynamic programming solution to compute F_n in *linear* time, and explain why it is correct.

2. **[10 pts]** What would it mean to add memoization to Strassen's matrix multiplication algorithm? What asymptotic improvement (if any) does it yield in the worst case? Explain your answer.

3. **[10 pts]** A subsequence is a sequence that can be derived from another sequence by deleting some elements without changing the order of the remaining elements; e.g. "acef" is a subsequence of "abcdef." Consider the problem of finding the longest common subsequence of two sequences – this is a task versioning systems like git or cvs often solve. Show that this is a special case of the sequence alignment problem. Then, give a polynomial-time algorithm for finding the longest subsequence common to *three* sequences. Analyze its running time and argue why it is correct.

4. **[10 pts]** You are given a complete binary tree on $n = 2^d - 1$ vertices (for $d \geq 1$), rooted at vertex r. Further, each vertex i in the tree is assigned a weight $w_i > 0$. The problem is to find the k-vertex subtree[1] (with $1 \leq k \leq n$) rooted at r for which the sum of the weights of its included vertices is maximized. Give an algorithm that does this in time polynomial in n and k and argue about its correctness.

[1] A subgraph of a graph G is another graph formed from a subset of the vertices and edges of G. The vertex subset must include all endpoints of the edge subset, but may also include additional vertices. A subtree is a connected subgraph of a tree.