STAT 473 – Game Theory
Spring 2020
Problem Set 1

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Due: 1/31/20, 9:30 am

1. [**10 pts**] Consider the game Chomp on a 3×3 board (see Figure 1). Draw a graph representation (as in Firgure 1.3 in the book) of all states reachable from the initial 3×3 position, and mark all edges as red, green, or black, as in the book. Mark each state as belonging to **N** or **P**.

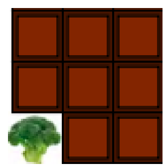

Figure 1: The starting position of a 3×3 game of Chomp

2. [**10 pts**] Consider the subtraction game with the subtraction set (i.e. the number of chips each player can remove on a turn) of $\{1, 2\}$. For which integers does the first ("next") player have a winning strategy? Argue for the correctness of your answer.

3. [**10 pts**] Prove that the first player in a game of (the usual 3×3) tic-tac-toe can always (at least) force a draw. *Note that one way to do this is to draw the full game tree, but there is an easier proof.*

4. [**10 pts**] Consider the following (silly) game. Players 1 and 2 play rock-paper-scissors *in turn* as a combinatorial game: first player 1 chooses among $\{R, P, S\}$, then (after seeing what player 1 has chosen) player 2 replies from among $\{R, P, S\}$. The payoffs (of $-1, 0$, or 1) to the two players are calculated using the usual rules (in Table 1 below. Draw the full (minimax) game tree of this game (see notes from Lecture 3 for some examples). What is player 1's payoff if both players play optimally?

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

Table 1: The payoff matrix for player 1. (The payoffs to player 2 are the negations of the payoffs to player 1)