

## HYPERBOLIC GLASS

For a fixed color of monochromatic light (e.g. as produced by a laser) and a given material, the light will move through the material at a particular speed, satisfying the relation  $v = c/n$ , where  $n$  is the **index of refraction** of the material, and  $c$  is the speed of light in a vacuum. To make a chunk of hyperbolic glass sitting on a table, make the index of refraction of the glass so that the speed of light changes in proportion to the height of the table, *i.e.* the index of refraction changes inversely proportional to the height above the table. Thus, light moves much more slowly near the table. This could be done by doping the glass at different heights with some other material, so that the index of refraction changes, as is done in certain types of fiber optics cables. When a laser is shined on the glass, the rays of light will be bent, and will follow paths which are circles or lines which are perpendicular to the table, as in figure 1. These paths are called **geodesics**. This is because light follows the shortest path. Since it moves more quickly higher above the table, to connect get from one point to another at the same height it is quicker to move upwards first where it can move more quickly, then come back down.

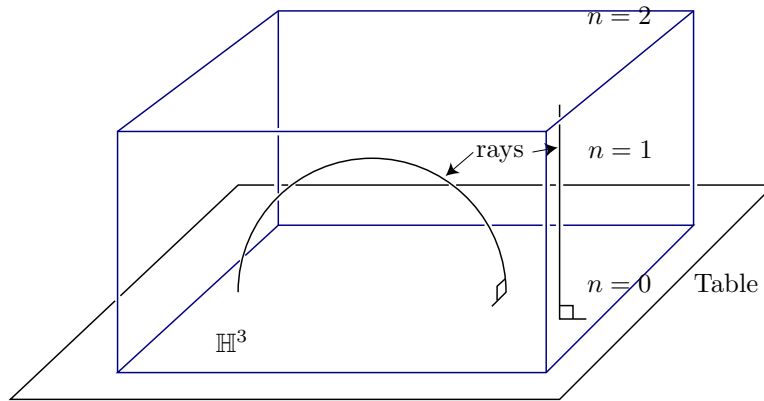


FIGURE 1. A block of hyperbolic glass

As the light gets closer to the table, it moves slower and slower, so that it never actually reaches the table. In practice, it would be impossible to make the glass so that its index of refraction approaches infinity. Instead, one could make a chunk which is a “hyperbolic tetrahedron”, where the edges are geodesics, the faces are sections of spheres which are perpendicular to the table, and dihedral angles are given in figure 2. Then one would mirror the sides, leaving little holes for viewing and for shining light into. The view would be like a hall of mirrors - you would see infinitely many copies of your eye, giving a physical model of hyperbolic space. The angles are unit fractions of  $2\pi$  so that when light is reflected around a corner, the image lines up. It would be cool if someone could actually make such a thing!

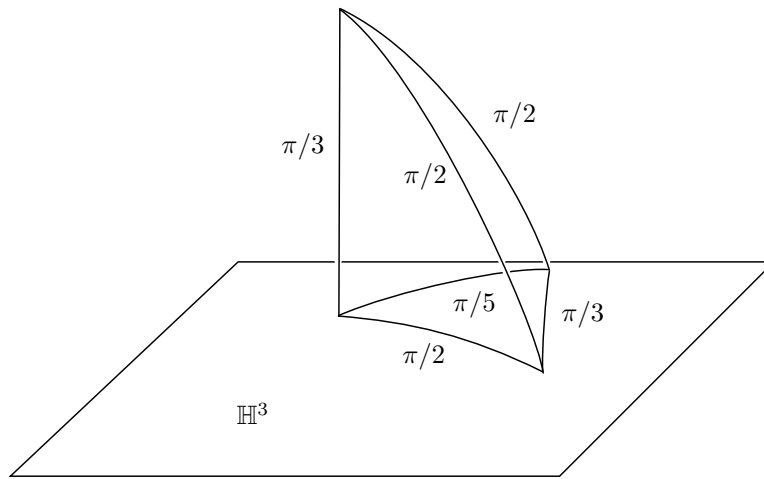


FIGURE 2. A tetrahedral chunk of hyperbolic glass