

Review of the lectures 15 to 28

On the exam you will be able to use Maple to verify your commands. No lecture notes or any other sources will be allowed. The questions below should help you reviewing the material. They are not model problems for the exam. Also review the homework assignments and quizzes.

1. Use `rand()` to generate a list of 100 numbers. Call the list 1.
 - (a) Replace every element x in the list 1 by $x \bmod 5$.
 - (b) Remove all duplicates from the list 1.
2. Explain why `piecewise` is often preferable to a similar `if-then-else` instruction. Give a good example to illustrate your explanation.
3. Write an indexed procedure with name `rf`, which returns `rf[n](x)`, defined by

$$\mathbf{rf}[0](x) = 1, \mathbf{rf}[1](x) = x, \text{ and } \mathbf{rf}[n](x) = (x + 1)(\mathbf{rf}[n - 1](x) - \mathbf{rf}[n - 2](x)), \text{ for } n \geq 2.$$

The index n is the degree of the polynomial. Make sure the recursion runs efficiently.

4. Divided differences for a function $f(x)$ are defined as follows:

$$f[x_1, x_2, \dots, x_{n-1}, x_n] = \frac{f[x_2, \dots, x_{n-1}, x_n] - f[x_1, x_2, \dots, x_{n-1}]}{x_1 - x_n}, \quad \text{for } n > 1$$

and $f[x_k] = f(x_k)$, for all k .

Write a Maple procedure `dvd` which computes divided differences for any `f`, and is called like `dvd[a, b, c, d](f)`. The output of `dvd[a, b, c, d](f)` shows

$$\frac{\frac{f(d)-f(c)}{c-d} - \frac{f(c)-f(b)}{b-c}}{b-d} - \frac{\frac{f(c)-f(b)}{b-c} - \frac{f(b)-f(a)}{a-b}}{a-c}.$$

For simplicity, assume the user always makes the correct call to `dvd`, i.e.: include no error handling features.

5. What is a remember table in Maple? How is it used? Mention an example of a good use of a remember table.
6. Use the arrow operator to define the following operations on a polynomial p :
 - (a) remove all terms with negative coefficients from p ;
 - (b) replace x by x^2 in $p(x)$.

Use these two functions to define a function which does both operations to a polynomial.

7. Create a function in the variables B and N which returns $B \sum_{k=0}^N r_k^k$, where r_k is a random number drawn from a normal distribution with mean five and standard deviation $0.1k$.

8. Explain the difference between symbolic and automatic differentiation. Illustrate with an example the difference between the two and give the two Maple commands you need.
9. What is the difference between `int` and `Int`?
Give a good illustration why we need a command like `Int`.
10. Consider the function $f(t) = \int_0^t (1 - e^x) dx$, for $t \geq 0$. Define this function in Maple.
What is $f'(0)$?
11. The function $g(x, t) = \frac{1-t^2}{1-2xt+t^2}$ is a generating function for the Chebyshev polynomials.
- Compute a Taylor series approximation for $g(x, t)$ around $t = 0$ of order 10. Select the coefficient of t^8 and compare with the output of `orthopoly[T](8, x)`. What is the difference between the two?
 - Make a function `cp` in n (n is the degree of the Chebyshev polynomial) which uses this generating function and returns the same expanded polynomial as the one returned by `orthopoly[T](n, x)`. The function `cp` should work for any n , be careful for $n = 0$.
12. Consider $p(x) = 5x^2a^2 + 61x^2a + 66x^2 + 10xa^2 + 121xa + 121x + a^2 + 15a + 44$, as a polynomial in x with parameter a .
- Find the roots of p .
 - For which values of the parameter a is the answer valid?
 - Give the Maple command(s) to treat the special case(s).
 - As you can see the polynomial p is shown in expanded form. Give the Maple command to “un-expand”, i.e.: what is the command which reveals better the structure of p ?
13. How would you best solve for x the following expression:
- $$-42 \sin(x)^{11} + 88 \sin(x)^8 - 76 \sin(x)^7 - 65 \sin(x)^5 + 25 \sin(x)^3 + 28$$
14. Let a and b be positive numbers. Consider $f = \frac{x^2}{a} + \frac{y}{b}$ and the unit circle $x^2 + y^2 = 1$.
Give all Maple commands . . .
- to determine the number of candidate extremal values of f on the unit circle.
 - to show how to compute one (only one!) such candidate extremal value.
15. The logarithmic spiral is defined by $r = ae^{bt}$ in polar coordinates.
- Give the Maple commands to make a plot for $a = 0.5$ and $b = 0.07$, for $t = 0 \dots 6\pi$.
 - Create an animation of 10 frames, for $a = 0.5$ and for b going from 0.01 to 0.1 (also for $t = 0 \dots 6\pi$).
16. Consider the curve $x^4 - 3xy + y^4$. Give all Maple commands

- (a) to make a plot for x and y both ranging between -2 and $+2$;
- (b) to convert the curve into polar coordinates; and
- (c) to plot the curve in polar coordinates.

17. Consider the system
$$\begin{cases} 4 + 2x + 2y + z + xy = 0 \\ 4 + 2x + 2y + 2z + 4y^2 = 0 \\ x + y + x^2 = 0 \end{cases}$$

- (a) How many complex solutions does the system have? Justify your answer.
 - (b) Find all complex solutions of the system.
 - (c) Suppose we were only interested in the rational or real solutions to the system. How would you modify your answer to the previous question to limit Maple to compute only the real or rational solutions?
18. Sometimes Maple displays symbols like `_C1`, `_C2`, etc... in its output.
- (a) What does this mean?
 - (b) Give a good example of a problem which would show symbols like `_C1`, `_C2`, etc...

19. Consider the initial value problem

$$x''(t) + 4x(t) = \sin(t), \quad x(0) = 1, x'(0) = 0.$$

- (a) Give all Maple commands to define this problem and to solve it numerically.
 - (b) Define a function which returns for every t the value of $x(t)$.
 - (c) Plot the solution for t going from 0 to 10.
20. Give the Maple commands for the following tasks.
- (a) Create a 5-by-5 matrix A where the (i, j) -the element is $\frac{1}{i+j}$ and a 5-by-1 matrix b of ones.
 - (b) Construct the augmented matrix $[A \ b]$ in order to solve the system $Ax = b$ with Gauss-Jordan elimination, i.e.: use `LinearAlgebra[ReducedRowEchelonForm]`.
 - (c) Compute a LU Decomposition of the matrix A and use `ForwardSubstitute` and `BackwardSubstitute` from the `LinearAlgebra` package to solve $Ax = b$.