Review of the first two parts of the course

In the first two parts of the course we learnt about Maple's advanced number system and about its favourite objects: polynomials and rational expressions.

Below is a list of questions to review. Consider also the review questions for the midterms, the quizzes, midterms, and homework assignments.

Review of Part One: First Steps with Maple

1. Explain the difference between `evalhf(1.0+10^(-10))` and `evalhf(1+10^(-10))`.

   The machine precision of a floating-point number system is defined as the smallest number you can add to one and obtain a result that is still larger than one. Give the Maple command(s) to get the magnitude of the machine precision.

2. Consider $\mathbb{Z}_{31} = \{ 0, 1, 2, \ldots, 31 \}$ and answer the following questions:

   (a) What is the multiplicative inverse of 7 in $\mathbb{Z}_{31}$?

   (b) Show that $p = 15x^5 + 4x^4 + 23x^3 + 26x^2 + 6x + 1$ is irreducible over $\mathbb{Z}_{31}$.

   (c) Declare α as a formal root of p. How many elements has $\mathbb{Z}_{31}(\alpha)$? Justify your answer.

   (d) Compute the value α^{21} as an element in $\mathbb{Z}_{31}(\alpha)$.

3. Give all Maple commands to write $e^{I\frac{2\pi}{k}}$ as $\cos(2\frac{\pi}{k}) + I\sin(2\frac{\pi}{k})$.

4. The sequence `restart; s := a+b: a := x+y: b := u+v: s;` shows `x+y+u+v`.

   (a) Give the Maple command to show that Maple still knows that $s = a + b$.

   (b) Give one single Maple command to change `s` so that typing `s` shows `x + y + u + v + c`.

5. Illustrate a good use of the **assign** command.

   Give an example of a Maple session in which the outcomes of **assign(x,5)** and **x := 5** are different.

6. Consider the expression $q = \cos(x^3 - 1) + 3\sin(y) - z^7$. Draw the expression tree for q and give all Maple commands you used to make the drawing.

7. How do we bring a matrix of floating-point numbers from file into a Maple session?
   Illustrate with a good example.

8. Generate optimized code to evaluate $p = 79x^{298} + 56x^{205} + 49x^{164} + 63x^{121} + 57x^{119} - 59x^{42}$.
   How many aritmetical operations are needed to evaluate p?
   Compare with the cost of a direct evaluation of p.

Review of Part Two: Polynomials and Rational Expressions

9. Draw the internal representation of $p := xy(x-y)$. Give also the Maple command(s) (but not the output!) used to obtain your drawing. Explain why **subs(1=-1,p)** returns $\frac{1}{xy(-x-y)}$.

10. Consider the polynomial $p = x^3 - x - 2$ and give all Maple commands following questions:

    (a) to write p as an **exact** product of linear factors, with exact complex numbers;

    (b) to compute a **numerical** factorization of p over the complex numbers;

    (c) to define a **symbolic** (i.e.: formal) factorization of p, declaring sufficiently many roots.

11. Give all Maple commands to transform $(x-y)(x+y)$ into $(x+y)x-(x+y)y$.

12. Consider the rational expression $r = \frac{79x^5+56x^4+49x^3+63x^2+57x-59}{45x^5-8x^4-93x^2+43x-62}$.

    Convert r into a form which is more efficient to evaluate. Compare the number of arithmetical operations needed to evaluate r in this more efficient form with the number of arithmetical operations needed to evaluate r in its given form.

13. Explain why normal forms are so important to symbolic computation.
    What can we do if a normal form is too expensive to compute? Illustrate with a good example.

FINAL EXAM is on Tuesday, May 5, 2009, 10:30 AM - 12:30 PM