Final Exam Material

Note: This is not an exhaustive list of problems. Please do not restrict your studying to this review sheet.

1. Consider the parallogram whose vertices are at $(0,0,0), (1,\sqrt{3},0), (1+\sqrt{3}, 1+\sqrt{3}, 0)$, and $(\sqrt{3},1,0)$.

   [i] Find all the interior angles in this parallelogram.

   [ii] Find the area of this parallelogram, first using vector arithmetic, then using a double integral.

2. Find a vector perpendicular to both $u = (1,2,1)$ and $v = (1,4,0)$.

3. Find the equation of the plane which contains the the origin and the points $(1,2,1)$ and $(1,4,0)$.

4. Find the equation of the plane $z = x$ in cylindrical, and also, in spherical coordinates.

5. Explain why $\mathbf{r}(t) = (t^2+1, t^2, t^4+1)$ is not the same curve as $\mathbf{r}(s) = (s+1, s, s^2+1)$.

6. Show that $\frac{d}{dt}\big(\mathbf{r}(t) \times \mathbf{r}'(t)\big) = \mathbf{r}(t) \times \mathbf{r}''(t)$ for any twice differentiable $\mathbf{r}(t)$.

7. Find the tangent vector to $\mathbf{r}(t) = (R\cos(t), R\sin(t), 0)$. Verify that the circumference of a circle is $2\pi R$, by finding the arclength of $\mathbf{r}(t)$ for $0 \le t \le 2\pi$.

8. Find the curvature of a circle of radius R, using the above parameterization.

9. What is the domain of the function $f(x,y) = \ln(x^2 - y^2)$. Find the level curve(s) corresponding to $f(x,y) = 0$.

10. Is the function $f(x,y) = \arctan(y/x)$ continuous at $(1,0)$? What about $g(x,y) = \frac{\sqrt{x^2+y^2}}{x+y}$ at $(0,0)$? And $h(x,y) = \frac{x^2y}{x^2+y^2}$ at $(0,0)$?

11. Show that $f(x,y) = \sqrt{x^2+y^2}$ is differentiable near $(1,0)$ but not near $(0,0)$.

12. Find the tangent plane to $f(x,y) = \exp(-x^2-y^2)$ at $(0,0)$.

13. Given a surface $z = f(x,y)$, how is the vector (2 components) ∇f related to the normal vector to the surface (3 components)?

14. In which direction is the function $f(x,y) = \frac{1}{4}x^3 + y^3$ at the point $(1,2)$ increasing the fastest? decreasing the fastest?

15. Consider a coordinate system $u(x,y) = 2x - y$ and $v(x,y) = 5x + 6y$, and the function $f(u,v) = u^2 - v^2$. Find $\frac{\partial f}{\partial x}$ at $x = 1, y = 2$.

16. Find the global maximum and minimum of $f(x,y) = x^4 - yx + \frac{1}{2}y$ in the domain bounded by $y = x^2, x = 0$, and $y = 1$.

17. Find the closest point to the origin which is on the hyperbola $xy = 2$.

18. Calculate $\int\int yx^2\sqrt{1+x^3}dA$ over the region bounded between $y = x^2 - 1$ and $y = 1 - x^2$.

19. Calculate $\int_0^1 \int_x^1 e^{-y^2} dydx$.

20. Find the volume of the solid bounded by $z = 0, z = 2x + y + 3, 0 < x < y$, and $0 < y < 1$.

21. Find the center of mass of the solid in the previous problem.

22. Find the volume of the region inside bounded by $x^2 + y^2 < 1, z < 1 - x^2 - y^2$ and $z > 0$. *Hint, use cylindrical coordinates.*

23. Find the volume of the region bounded by $x^2 + y^2 > z^2, x^2 + y^2 + z^2 < 1$, and within the first octant.

24. Find the Jacobian of the change of variables $x = u^2 + v^2$, and $y = u^2 - v^2$.

25. Evaluate the $\int_C \mathbf{F} \cdot d\mathbf{s}$ for $\mathbf{F} = (x, y^2x)$ with C the lines which connect $(0, 0)$ to $(1, 0)$ then $(1, 0)$ to $(1, 3)$.

26. Show that $\mathbf{F} = (x^3, y^3, z^3)$ is a conservative vector field.

27. Find a potential for $\mathbf{F} = (x^3, y^2, z)$, then evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$, where C connects $(1, 2, 4)$ to $(3, 1, -1)$.

28. Find the surface area of the cone $z = 1 - \sqrt{x^2 + y^2}$, above $z = 0$.

29. Evaluate the line integral of $\mathbf{F} = (y, -x)$ over the ellipse parameterized $x(t) = 2\cos(t)$ $y(t) = -4\sin(t)$, with $0 \leq t \leq 2\pi$.