[Floating Point] A floating point system with a three bit mantissa and four bit exponent, represents a number as

$$1.d_1d_2d_3 \times 2^{e_1e_2e_3e_4-s}$$

Where $d_i = \{0,1\}$ and $e_i = \{0,1\}$ and $s = 111$, all base 2. Convert the largest number in this binary system to a decimal. Find ϵ, the distance between 1 and the next number larger than 1.

[Bisection] Use bisection to calculate $\log_{10}(2)$ to two decimal places (by hand).

[Fixed Point] Find the convergence rate of the fixed point method when applied to the function $f(x) = x^3 - 3x^2 + 3x$, at all of its fixed points.

[Newton's Method] Show that Newton's method converges quadratically when $f'(p) \neq 0$.

[Secant Method] The secant method is being applied to the function $f(x) = x^3 - 3x^2 + 3x$. If $x_1 = 1/2$, $x_2 = 2$, find x_3.

[Regula Falsi] Under what condition(s) does the Regula Falsi Method converge to a root of $f(x)$?

[Determinants] Find the determinant of A.

$$A = \begin{pmatrix} 0 & 3 & 4 \\ 2 & 6 & 1 \\ 1/3 & 1 & 1/2 \end{pmatrix}$$

[Solvability] If the determinant of $A = 0$, when does Ax=0 have a solution?

[Gaussian Elimination] Solve Ax=b, with A from the determinant problem, and $b = (1, 0, 2)$.

[Pivoting] Use gaussian elimination with a pivoting scheme which reduces loss of precision error to solve

$$\begin{pmatrix} eps & 2 & eps \\ 1 & eps & eps/2 \\ 2 & 1 & 3/eps \end{pmatrix} x = \begin{pmatrix} 2 \\ 1 \\ 3/eps \end{pmatrix}$$

[LU Decomposition] Find the $PA = LU$ decomposition of A from the determinant problem.

[Vector Norms] Calculate $||x_i||_1$ and $||x_i||_\infty$ for

$$x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \text{and} \qquad x_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

[Matrix Norms] Antisymmetric matrices have $S_{ij} = -S_{ji}$. Show that $||S||_1 = ||S||_\infty$

[Iterative Methods] Consider an iteration $x^{k+1} = Mx^k + f$, which converges in $n/4$ steps. Which method finds $x = A^{-1}b$ with a smaller operation count, gaussian elimination (with LU decomposition) or this iteration? What if we want to solve this problem for three different vectors b

[Jacobi] Write the Jacobi method. Find the eigenvalues of the iteration matrix when this scheme is applied to a general 2×2 matrix A.

[Gauss Seidel] When does the Gauss-Seidel method converge when applied to an antisymmetric matrix A.

[SOR] Write the SOR method. To what does this method reduce when $\omega = 1$? $\omega = 0$?

[Lagrange polynomial] Find the lagrange polynomial through $(5, 0), (0, 5)$, and $(2, 2)$.

[Newton's divided difference polynomial] Use Newton's divided difference and your polynomial from the previous problem, to find a polynomial through $(5, 0)(0, 5)(2, 2)$, and $(-1, 1)$.

[Error of polynomial interpolation] Show that if $f^n(x) = 0$ for all x, then the polynomial which agrees with $f(x)$ at n points agrees with $f(x)$ everywhere.

[Runge Phenomenon] Discretize [-1,1] with five chebyshev points.

[Least Squares] Find the parabola which best fits the points $(1, 0), (2, 1), (3, 3)$, and $(4, 5)$.

[Numerical Differentiation] Find the truncation error of the approximation $f'(x, h) \equiv \frac{1}{h}(f(x + 3h) - 2f(x + h) + f(x))$

[Undetermined Coefficients] Find a third order scheme

$$f'(x, h) \equiv af(x) + bf(x + h) + cf(x + 2h) + df(x + 3h)$$

[Richardson Extrapolation] Use Richardson extrapolation and the scheme from the previous problem to find a fourth order scheme.

[Trapezoidal Rule] Derive the error formula for the trapezoidal rule.

[Simpson's Rule] Derive the coefficients for Simpson's rule by integrating a quadratic over an interval of width $2h$.

[Gaussian Quadrature] Derive the weights and points for gaussian quadrature when two points are used to discretize the interval $(-1, 1)$.

[Stability] Find the linear stability region of the scheme

$$u(t+k) - u(t) = kf(u(t))$$

[Consistency] Find the truncation error, when approximating $u'(t) = f(u)$, of the scheme

$$u(t+k) - u(t) = k/2\{f(u(t+k)) + f(u(t))\}$$