To solve $Ax = b$, we have learned three iterations. In all cases, the matrix A is decomposed into a diagonal part D, a strictly lower triangular part L, and a strictly upper triangular part U.

The Jacobi-iteration

$$x^{k+1} = -D^{-1}(L+U)x^k + D^{-1}b$$

The Gauss-Seidel iteration

$$x^{k+1} = -(D+L)^{-1}Ux^k + (D+L)^{-1}b$$

The Successive Over Relaxation (SOR) iteration

$$x^{k+1} = (D+\omega L)^{-1}((1-\omega)D - \omega U)x^k + \omega(D+\omega L)^{-1}b$$

Notice that when $\omega = 1$ the SOR iteration reduces to the Gauss-Seidel iteration.

Convergence: A sufficient condition for convergence of a method is the iteration matrix has $||M|| < 1$. A necessary and sufficient condition is that $\rho(M) < 1$. For the Gauss-Seidel method and the Jacobi iteration we also know that the iterations converge if the original matrix A is diagonal dominant (the sum of the magnitudes of the off-diagonal entries in a row is less than the magnitude of the diagonal entry).