Name:____

U.I.N. :____

0.1 Analysis:

For the problems that follow, all computations should be done "by hand."

1. **Binary Representation** Write the following base 10 numbers as their binary representations.

   [i] 27

   [ii] 15.140625

2. **Floating Point Arithmetic** *For these exercises assume that numbers are expressed in the form* $r = \pm 0.d_1 d_2 \ldots.d_k \times 10^t$*, with* $k = 5$ *and* $t \in [-5, 5]$*). The* d_i *are integers in* $[0, 9]$.

   [i] What is the largest number that can be expressed in this system? the smallest in magnitude?

   [ii] If chopping is used (rather than rounding), the number $.12300 \times 10^3$ represents what interval of real numbers.

3. **Fixed Point Algorithm**

   [i] Show that the fixed point algorithm $x_{n+1} = f(x_n)$ converges *locally* when $|f'(x)| < k < 1$ in the neighborhood of the fixed point.

   [ii] What conditions must the function $f(x)$ satisfy for the fixed point iteration $x_{n+1} = f(x_n)$ to converge faster than linearly.

4. **Newton's Method**

   [i] Show that Newton's method converges quadratically to a root p of a twice differentiable function f, if $f'(p) \neq 0$ and $f''(p) \neq 0$.

   [ii] Does Newton's method ever converge faster than quadratically? Justify your answer by writing the appropriate power series expansions. What does this imply about the associated fixed point method?

0.2 Numerics

For the problems that follow, original computer programs should be used to answer the questions. Attach your code, but keep it separate from your solutions.

5. Write a program to find roots of a function using the bisection method, Newton's Method, and the secant method.

[i] Apply your program to find the two roots of $f(x) = \cos(x)\exp(-x^2)$ nearest the origin.

[ii] Plot the log of the error at the nth iteration against the log of the error at the (n+1)st iteration for each scheme on the same axis. What conclusion can be made from this graph?

[iii] For each method, try finding the basin of attraction of the root $p = \pi/2$. One way to approach this is to discretize an interval about $\pi/2$, for example $[-3, 3]$, and use every point of your discretization as an initial guess. For Newton's method you will get intervals where the method converges. For the secant method and the bisection method you will have two initial guesses, so the result will be a region in the plane where the method converges. Include plots of your computed basins for each method with your homework.