Name:_____

U.I.N. :_____

Analysis:

For the problems that follow, all computations should be done "by hand."

1. Find the the polynomial of degree 4 which passes through $(1,2), (-1,6), (2,8)(3,2)$ and $(0,7)$ using Newton's divided difference approach.
2. Find the line which best fits $(1,0), (2,3), (4,5), (2,0), (1,1), (-1,-2)$, with error measured in the the vector 2 norm.
3. Find the truncation error for the leapfrog difference discretization of the derivative
$$F_h(x) \equiv \frac{f(x+h) - f(x-h)}{2h}.$$
4. Find the best step size for the above difference method when both truncation and rounding errors are considered.

Numerics:

For the problems that follow, original computer programs should be used to answer the questions. Attach your code, but keep it separate from your solutions.

5 Observe the error of a polynomial interpolant by plotting the function $E(x) = C\prod(x - x_i)$, with $C = 1$, for

[i] $\{x_i\}$ nine equally spaced points in $[-1, 1]$

[ii] $\{x_i\}$ nine Chebyshev points in $[-1, 1]$

6 Implement the leapfrog difference scheme of problem 3. Plot the log of the error against the log of the step size, h, when applied to $f(x) = x^3$, at x=1, as $h \to 0$. Label the step size where the truncation error becomes smaller than the rounding error.

7 Write a code which implements both the midpoint rule and Simpson's rule (with weights 1 4 2 4 ... 2 4 1) for evaluating integrals. Plot the log of the error as a function of the log of h (the step size) when approximating $I_1 = \int_0^\pi \sin(x)$ and $I_2 = \int_0^2 x$.