

**Exam 1 Material**

**Note:** This is not an exhaustive list of problems, nor is it in the format of the exam. It is just some practice problems to act as a study aid.

For the problems 1-6 use  $\vec{v} = (1, 6, 2)$  and  $\vec{w} = (3, 0, -1)$

1. Draw  $\vec{v}, \vec{w}, \vec{v} + \vec{w}$ , and  $\vec{v} - \vec{w}$ .
2. Find the equation(s) for the line in the direction of  $\vec{v} + \vec{w}$  and through the point  $(3, 4, 5)$ .
3. Find the angle between  $\vec{v} + \vec{w}$  and  $\vec{v} - \vec{w}$ .
4. Find a unit vector perpendicular to both  $\vec{v}$  and  $\vec{w}$ .
5. Find the equation for the plane which contains the endpoints of  $\vec{v}$  and  $\vec{w}$  (including the origin).
6. Find the projection of  $\vec{v}$  onto  $\vec{w}$ . How is this different from the component of  $\vec{v}$  in the direction of  $\vec{w}$ .
7. If  $\|\vec{u}\| = 3$ ,  $\|\vec{q}\| = 2$ , and  $\theta = \pi/6$  where  $\theta$  is the angle between  $\vec{u}$  and  $\vec{q}$  compute

$$\frac{\|\vec{u} \cdot \vec{q}\|}{\|\vec{u} \times \vec{q}\|} =$$

8. Find the equation(s) for the surface  $z^2 = x^2 + y^2$  in both cylindrical coordinates and spherical coordinates.
9. Parameterize the curve formed by the intersection of the cylinder  $x^2 + y^2 = 4$  and the sphere  $(x - 2)^2 + y^2 + z^2 = 1$ .

10. For  $\mathbf{r}(t) = \langle 2t^2 - 1, \ln(t), \sqrt{t} \rangle$ , calculate:

The tangent vector  $r'(t)$ ,

The tangent vector  $T(t)$ ,

The unit tangent's derivative  $T'(t)$ ,

The curvature at  $t = 4$ ,

$T' \cdot T$  and  $T \times T$ .

The instantaneous rate of change of the arclength at  $t = 4$ .

11. Calculate the following limits (stating whether they exist, with justification)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\tan(x)e^y \cos(y)}{xe^x}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^3 + y}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x) \sin(\frac{1}{y})y}{x}$$

12. For the function  $f(x, y) = \sqrt{x^2 + y^2}$

Find  $f_x$  and  $f_y$ ,

Convert  $f_x$  and  $f_y$  to polar coordinates,

Does partial derivative exist at  $(x, y) = 0$  (or  $r = 0$ )?