Exam 2 Material

Note: This is not an exhaustive list of problems, nor is it in the format of the exam. It is just some extra practice problems.

1. Where is the function $f(x, y) = \sqrt{x^2 + y^2}$ continuous? Where is it differentiable? Find the linear approximation (tangent plane) to f at a point where it is differentiable.

2. Find the directional derivative of the function $f(x, y) = \cos(x)\sin(y)$ at the point $(\pi/3, \pi/4)$ in the direction $(\sqrt{6}, \sqrt{2})$.

3. Does the existence of a gradient at a point imply the existence of directional derivatives at that point? What about the reverse?

4. For (x, y, z) on the surface $F(x, y, z) = 0$, with $F(x, y, z)$ differentiable, show that
$$\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} = -1$$

5. Calculate $\frac{\partial}{\partial s}F(x(s), y(s))$ if $\frac{\partial F}{\partial x} = 5$, $\frac{\partial F}{\partial y} = 3$, $\frac{\partial x}{\partial s} = -1$ and $\frac{\partial y}{\partial s} = 2$.

6. Find and classify all critical points of $F(x, y) = x^3 y - x^2 - y^2$.

7. Find the maximum of $F(x, y, z) = x^2 - y - z$ subject to the constraint $x^2 - y^2 + z = 0$.

8. Evaluate
$$\int_0^3 \int_2^4 x^2 y^2 dx dy$$

9. Find the new bounds if $\int_{-2}^{2} \int_0^{4-x^2} f(x, y) dy dx$ is written in the form $\int \int f(x, y) dx dy$.

10. Find the average value of $F = \sqrt{x^2 + y^2}$ over the unit disc.

11. Find the center of mass of the solid described by $0 < x < 1$ and $x^2 + y^2 + z^2 \leq 1$ with density function $\rho(x, y, z) = x^2 + y^2 + z^2$.

12. Evaluate the volume of the region which is above the plane $z = 0$, outside of the cone $z = \sqrt{x^2 + y^2}$, and inside of the sphere $x^2 + y^2 + z^2 = 1$

    [i] Using an integral in cylindrical coordinates.

    [ii] Using an integral in spherical coordinates.

13. Consider the change of variables $u(x,y) = x^2 - y^2$, $v(x,y) = x^2 + y^2$, find the Jacobian of this transformation.

14. Draw a representation of the vector field which corresponds the potential $\phi = e^x$.

15. Calculate $\int_C \mathbf{F} \cdot d\mathbf{s}$ if C is the curve which connects $(0,1)$ to $(1,5)$ via $x(t) = t$ $y(t) = t^2 + 1$ with $t \in [0,1]$, and $\mathbf{F} =< x, \sqrt{y} >$.