

Let  $f(x_1, \dots, x_k)$  be a polynomial over  $\mathbb{R}[[t]]$ . If  $f$  has a zero modulo  $(t^n)$  for every  $n$  then  $f$  has an actual zero in  $\mathbb{R}[[t]]^k$ . The notion of *extremality* may be seen as a generalization of this fact to arbitrary valued fields. A valued field  $K$  is extremal if the values of every polynomial  $f(x_1, \dots, x_k)$  over  $K$  reaches a maximum (possibly  $\infty = v(0)$ ) when evaluated at tuples from the valuation ring.

Henselian valued fields were expected to be extremal, when residue characteristic is 0. We present a hands on account of how this expectation fails and provide a surprising classification theorem for extremal valued fields.