Math 430: Formal Logic I. Propositional Logic Review.
Based on Chapter 2 and Appendix C of our textbook.

For the duration of this review: Let A_i be propositional variables for $i \in \mathbb{N}$. Let α, β, ϕ, ψ, and θ be propositional formulae all of whose propositional variables are A_i for some $i \in \mathbb{N}$. Let e be a truth evaluation that assigns 0 to A_i for odd i, and 1 to A_i for even i.

Part 1: True/False/Nonsense

For the next six questions, circle T for true, F for false, and N for nonsense. For example, "$2 = 3$" and "2 is even" are true; $2 \in \{1, 3\}$ and $0 \sim (A_1 \leftrightarrow A_1)$ are false; while $2 \sim 1$ is nonsense (2 isn't a propositional formula), and "3/5 is even" is nonsense (only integers can be even).

1. $(A_1 \wedge A_2) = 0$
   T F N
2. $(\widehat{\alpha \vee \beta})(e) = (\widehat{\beta \vee \alpha})(e)$
   T F N
3. $\widehat{0}(e) = 0$
   T F N
4. $((A_1 \to (\neg(A_2) \to A_1)) \to A_2) \sim ((\neg A_2) \to (A_1 \wedge (\wedge(A_2 A_1) \wedge A_1)))$
   T F N
5. $(\alpha \widehat{\vee \beta \wedge} A_1)(e) = (\beta \widehat{\vee \alpha \wedge} A_1)(e)$
   T F N
6. $((A_1 \wedge A_2) \to A_3) \sim ((A_1 \to A_2) \to (A_1 \to A_3)$
   T F N

Part 2: True/False

For each of the following, decide whether the statement is true or false:

1. If $(\alpha \vee \beta)$ is a tautology, then either α or β is a tautology.
   T F
2. If $(\alpha \vee \beta)$ is a contradiction, then either α or β is a contradiction.
   T F
3. A, β, $\therefore (A \to \beta)$ is a valid arguement.
   T F
4. If $\{\alpha\}$ is satisfiable and $\{\beta\}$ is satisfiable, then$\{\alpha, \beta\}$ is satisfiable.
   T F
5. If e satisfies α and e also satisfies β, then e satisfies $\{\alpha, \beta\}$.
   T F

6. If α is a tautology, then $\hat{\alpha}(e) = 1$.
   T F

7. A ternary connective t whose truth table is the truth table for the formula $(\neg((A_1 \vee A_2) \wedge A_3))$ is adequate.
   T F

Part 3: Short answers

1. Compute the truth table for the formula $((A_1 \wedge A_3) \leftrightarrow \neg(\neg A_1 \rightarrow A_2))$. Circle the row corresponding to e.

2. Suppose that $((P \vee (Q \vee P)) \vee (Q \vee (P \vee (Q \vee P))))$ is the result of substituting α for every occurence of P in $(P \vee (Q \vee P))$. What is α?

3. Translate the following argument into symbolic form and determine whether it is valid:
   If Bobby has red eyes and doesn't want to play, then either Bobby is sick or he has been crying.
   If Bobby is sick, then He has been crying.
   Bobby's eyes are red.
   Therefore, if Bobby doesn't want to play, he must have been crying.

4. Find a propoitional formula $\phi(A, B, C)$ with the following truth table:

A	B	C	ϕ
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0