

Math 430: Formal Logic I
1st homework set, due friday, september 5th.

Bring your solutions to class on friday, or slide them under the door of SEO716.

This homework is based on the first week of lectures, roughly corresponding to Sections 2.1 and 2.2 and appendix C of our textbook. For this homework, please do **not** use abbreviations of formulas, but write them out with all the parentheses. The last question is not a joke. Good taste in interesting questions is important to a mathematician.

1. Consider the statement “The moon is made of green cheese if and only if I am not the princess of Monaco.”
 - (a) Translate this into a propositional formula. (Start by identifying the basic parts of the statement and picking a propositional variable for each one.)
 - (b) Make a truth table for this formula.
 - (c) Which truth evaluation (i.e. which row of your truth table) corresponds to reality, to the best of your knowledge? Is the statement true?
2. Let D be the propositional variable standing for “my dog ate my homework”, let S be the propositional variable standing for “my shredder ate my homework”, and let B stand for “I deserve a bad grade on this homework”. Write down an english sentence corresponding to $((D \vee S) \wedge ((D \vee S) \rightarrow (\neg B)))$.
3. We define a function va that takes propositional formulae and returns sets of propositional variables by induction as follows:
 - (Base Case) $va(0) = va(1) = \emptyset$ and $va(x) = \{x\}$ for each propositional variable x . (for example, $va(A) = \{A\}$.)
 - (Induction Step) For any two formulae ϕ and ψ , $va(\neg\phi) = va(\phi)$ and
$$va((\phi \vee \psi)) = va((\phi \wedge \psi)) = va((\phi \rightarrow \psi)) = va((\phi \leftrightarrow \psi)) = va(\phi) \cup va(\psi)$$

Compute

- (a) $va((A \vee B))$
 - (b) $va(((A \vee B) \wedge ((\neg C) \vee B)))$
 - (c) $va(((\neg(D \vee S) \wedge ((D \vee S) \rightarrow \neg B)) \vee ((A \vee B) \wedge ((\neg C) \vee B))))$
4. Prove that for any two formulae α and β , the formula $(\alpha \rightarrow \beta)$ is a tautology if and only if for every truth evaluation e , if $\hat{\alpha}(e) = 1$, then $\hat{\beta}(e) = 1$.
 5. Ask an interesting question and try to answer it.