Math 430: Formal Logic I
11th and last homework set, due monday, december 1st.
Bring your solutions to class, or slide them under the door of SEO716.

This homework is based on sections 5.1-5.8 of our textbook.

1. Excercises from pp.328-329 of the textbook, about natural numbers:
   (a) 5.3.1(a)
   (b) 5.3.2(a)
   (c) 5.3.3(a)
   (d) 5.3.3(e)
2. Write a careful, detailed proof of Proposition 5.11.2 on p.352 of our textbook.
3. Suppose that the variables $x_1, x_2, \ldots x_n$ in an L-term t do not occur in an L-formula $\phi(y)$; let $\psi(x_1, x_2, \ldots x_n)$ be the L-formula obtained by replacing each occurrence of y in ϕ by t; then $\alpha := \forall y\, \phi$ and $\beta := \forall x_1 \forall x_2 \ldots \forall x_n\, \psi$ are L-sentences.
   (a) Is $\alpha \therefore \beta$ a valid argument?
   (b) Is $\beta \therefore \alpha$ a valid argument?
   (c) Dropping the assumption that variables x_i do not occur in ϕ, find a formula ϕ and a term t such that both of the above are not valid.
4. Axiomatize the the class of algebraically closed fields. That is, in the first-order language with two binary function symbols $+$ and $\cdot$ and two constant symbols 0 and 1, write an infinite list S of first-order sentence so that any L-strucutre that satisfies all sentences in S is an algebraically closed field, and all algebraically closed fields satisfy all sentences in S.
5. Think back on everything we've learned this semester, ask an interesting question and try to answer it.