Ma502: Metamathematics I
1st homework set, due wednesday, september 12th.
Chapter 2 of Ebbinghaus, Flume, and Thomas (EFT).
Bring your solutions to class, or slide them under the door of SEO716.

1. For this problem the symbol set is $\{R, \text{bob}, \boxdot\}$, where R is a ternary relation symbol, *bob* is a constant symbol, and $\boxdot$ is a unary function symbol. For each of the following, say whether it is a **T**erm, a **F**ormula, an **I**nitial segment of a term or a formula, or **N**one of the above. For terms and formulae, write out the derivation in the apprpriate calculus (like in examples on pages 16 and 17 of EFT), or draw the composition tree like we have done in class.

   (a) $R(v_0, v_1 7)$
   (b) $\boxdot \boxdot \boxdot v_1$
   (c) $\forall v_3 R v_3 \boxdot v_2 \boxdot \boxdot v_3$
   (d) $(R\text{bobbobbob} \wedge \text{bob} \equiv v_0)$
   (e) $\exists v_0 \text{bob} R \vee$

2. For this problem, the symbol set contais one binary relation symbol $<$. Translate the following english expressions into formulae, or explain why you're having trouble doing so. (At this point, you don't have tools to *prove* that something cannot be expressed by a formula.)

   (a) One number is less than another number but greater than the third number.
   (b) $<$ is a (partial) ordering, meaning that it is transitive (if one thing is less than another, and that other is less than the third, then the first is less than the third) and irreflexive (nohting is less than itself).
   (c) (well-ordering) There are no infinite decscending chains, i.e. sequences of elements of the universe where the second is less than the first, the third is less than the second, and so on.
   (d) (total ordering) Every two elements are comparable, meaning that one is less than the other, or the other is less than the first, or they are equal.
   (e) (complete total ordering) $<$ is a total ordering, and every subset that has an upper bound also has a least upper bound.

3. Read the proof of Lemma 4.2 in EFT.

4. We say that ϕ is a *subformula* of ψ if ϕ occurs in some derivation (equivalently, composition tree) of ψ.

   (a) Do we get the the same definition if we replace "some" by "every"?

   (b) How is this notion related to SF in definition 4.5.b on p.23 of EFT?

   (c) Suppose that ϕ is a subformula of ψ. Show that $\text{var}(\phi) \subseteq \text{var}(\psi)$. (var is defined in 4.5.a on p.23.)

   (d) Suppose that ϕ is a subformula of ψ. What is the relation between $\text{free}(\phi)$ and $\text{free}(\psi)$? (free is defined in 5.1, p.25.)

5. Ask an interesting question and try to answer it.