

Ma502: Metamathematics I

11th and last homework set, due wednesday, november 28th.

Chapter 10 of Ebbinghaus, Flume, and Thomas (EFT) and Chapter 3 of Kaye.

Bring your solutions to class, or slide them under the door of SEO716.

1. Exercise 6.13 on p.181 of EFT.
2. Use Theorem 4.1 on 167 of EFT to do exercise 4.3 on p. 170 of EFT.
3. Exercise 3.5 on p.40 of Kaye (warning: may be difficult)
4. Fix a first-order language L . For an L -theory T , let

$$\text{Mod}(T) := \{\mathcal{M} \mid \mathcal{M} \models T\}$$

be the class of L -structures satisfying T . For a class K of L -structures, let

$$\text{Th}(K) := \{\phi \in L \mid \forall \mathcal{M} \in K \mathcal{M} \models \phi\}$$

be the set of sentences true in all structures in K .

- (a) Show that if $T \subset T'$, then $\text{Mod}(T) \supset \text{Mod}(T')$.
 - (b) Show that if $K \subset K'$, then $\text{Th}(K) \supset \text{Th}(K')$.
 - (c) Is $\text{Th}(\text{Mod}(T)) = T$? Is $\text{Mod}(\text{Th}(\text{Mod}(T))) = \text{Mod}(T)$?
5. Ask an interesting question and try to answer it.