

Math 507: Stability

6th and last problem set, due wednesday, december 2nd.

1. Show that a stable ring without zero-divisors is a skew-field.
Hint: actually, show that a stable monoid with cancellation is a group.
2. Show that a type that is generically closed under the group operation is the generic type of its stabilizer. That is, suppose that G is a sufficiently saturated model, and that $p \in S(G)$ is a type such that for any two independent (over G) realizations a, b of p , the product ab is also a realization of p ; and prove that p is the generic type of the partial-type-definable group $\text{Stab}_G(p)$.
3. Find a right-generic but not left-generic subset of a (necessarily unstable) group.
4. Is it true that any type is a translate of the generic type of its stabilizer?
5. Suppose that H is a nice group and R is a nice' ring. For what values of nice and nice' (stable, superstable, ω -stable, strongly minimal, finite, etc) is the group ring $R(H)$ a stable group?
6. Prove that in a stable group $U(p) = R^\infty(p)$ for all types p , or produce a counterexample.