Math 180: Calculus I: Thinking problems.

Think about these problems, talk to other students about them, and try to understand what is happening - that's what mathematicians do. Feel free to play with them numerically or use calculators and computers, but that will only give you *guesses*, not explanations. Write down your observations and explanations carefully, in complete sentences. Please do not collaborate on writing - only on thinking and investigating. Unlike the webassign homework, these will be graded by a human being, who will look for correct explanations and clear writing. Guesses and numerical observations will not recieve credit - they are only investigative tools.

1. (Assigned Friday, September 4th, due Thursday, September 10th.)

   Explore the function $f(x) = sin(e^{\frac{1}{x}})$ near $x = 0$. Does $\lim_{x\to 0} f(x)$ exist? What about one-sided limits? If the limit does not exist, describe in words what the function does and try to prove, as rigorously as you can, that it does not exist. What does $f(x)$ do as x tends to ∞? as x tends to $-\infty$? Sketch a graph of this function.

2. (Assigned Friday, September 11th, due Thursday, September 17th.) Consider the function

   $$g(t) := \frac{x^3 - 7x + 6}{|x-1| \cdot (x-2)}$$

   For each of the following intervals

   $$[-8, -2], [-3, 0.5], [0.5, 1.5], [1.5, 3]$$

   explain whether the Intermediate Value Theorem can be applied to find a solution of the equation $g(t) = 2$ inside that interval. What about a solution of $g(t) = 5$? (So you should have eight yes/no answers, each carefully explained.)

3. (Assigned Friday, September 18th, due Thursday, September 24th.) Consider the function $R(z) = z \cdot |z|$. Find $\frac{dR}{dz}$. What is the domain of $\frac{dR}{dz}$? Explain your work!

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4. (Assigned Monday, October 5th; due Thursday, October 8th.) Suppose that you have two functions, $f(x)$ and $g(x)$, and you know that $f(x) = 2 \cdot g(x)$ for all x, and that $g(x)$ is differentiable everywhere and has a two-sided inverse $h(y)$.

   (a) Express $f^{-1}(y)$ in terms of $h(y)$. Check that your expression works for the special cases $g(x) = x^3$ and $g(x) = 3x$.

   (b) Which of the following sets of information are sufficient to compute $\frac{d}{dy} f^{-1}(y)$ at $y = 4$?

      i. $f'(1) = 2$ and $g(1) = 2$
      ii. $f'(1) = 4$ and $g(1) = 4$
      iii. $g'(1) = 2$ and $f(1) = 2$
      iv. $g'(1) = 4$ and $f(1) = 4$

      Explain your answers!

   (c) For the case(s) above where sufficient information is provided, compute $\frac{d}{dy} f^{-1}(y)$ at $y = 4$.

5. (Assigned Friday, October 23rd; due Thursday, October 29.)
   Sketch the graph of $f(x) = \ln(18 - (4 - x^2)^2)$. One hint: What is the domain of this function? Another hint: use the information about $f'(x)$ in looking for the zeros of $f''(x)$.

6. (Assigned Monday, October 26; due Thursday, November 5.)
   Find the right triangle with perimeter 7 with the greatest possible area. (Hint: being clever in several places can save you much ugly computation.)

More problems will appear here as the semester goes on.