

MATH 320: HOMEWORK 2

Due on Monday, September 9

Do problems 1.9, 1.12, 2.10 and 2.16 in the assigned Hefferon textbook.

1) Use induction to prove the following identity:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}.$$

2) Give a detailed proof that row equivalence is an equivalence relation on the set of all $m \times n$ matrices. Please include a careful definition of what it means for a matrix A to be row equivalent to another matrix B .

3) Suppose A and B are 7×7 matrices in echelon form such that A has 4 free variables and B has 6 free variables. Prove or disprove the following:

- (1) The associated homogeneous linear system to A has a unique solution.
- (2) A and B are row equivalent.
- (3) B has 5 free variables in row reduced echelon form.

Note, if you use a theorem or result from the text in your argument, please explicitly reference the statement.