MATH 320: HOMEWORK 5

Due on Friday, October 25

1) Find the determinant of the following matrices:

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 7 & 2 \\ 1 & 1 & 7 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 13 & 4 \\ 2 & 1 & 4 & 7 \\ 1 & 1 & 7 & 2 \\ 0 & 2 & 4 & 1 \end{pmatrix}.$$

2) Let $L : \mathbb{R}_1[t] \to \mathbb{R}_1[t]$ be the linear map defined by

$$L(a_0 + a_1 t) = a_0 + a_1 + a_0 t.$$

Consider the two bases $\mathcal{B} = \{1, t\}$ and $\mathcal{B}' = \{\sqrt{2} + t, e^{\pi} + t\}$. Compute the determinant of $[L]_{\mathcal{B}}$ and $[L]_{\mathcal{B}'}$.

3) Let V be a finite dimensional vector space. Let $L : V \to V$ be a linear transformation. Suppose $\mathcal{B}$ and $\mathcal{B}'$ are two bases of V. Prove that

$$\mathrm{Det}([L]_{\mathcal{B}}) = \mathrm{Det}([L]_{\mathcal{B}'}).$$

4) Suppose A is an $n \times n$ matrix such that Det(A) $= 0$. Consider the matrix

$$B = A^4 + 3A^2 + 73A.$$

Compute $\mathrm{Det}(B)$.

5) Use the adjugate matrix to compute the inverse of

$$\begin{pmatrix} 1 & 2 & 6 & 4 \\ 2 & 8 & 4 & 7 \\ 1 & 2 & 7 & 1 \\ 0 & 2 & 1 & 1 \end{pmatrix}.$$

6) An $n \times n$ matrix A is *skew symmetric* if and only if $A = -A^t$, where A^t is the transpose of A. If A is a skew symmetric matrix such that $\mathrm{Det}(A) \neq 0$, prove that n is even.

Date: