PRACTICE EXAM

Due on Friday, September 27

1)

(1) Find all solutions of the linear system

$$x + y - z = 3$$
$$x - y - z = 1.$$

(2) Find all solutions of the homogeneous system

$$x + y - z = 0$$
$$x - y - z = 0.$$

(3) Write down a basis for the vector space of solutions to the homogeneous linear system above. What is the dimension of this vector space?

2) Using elementary row operations, put the following matrix A in row reduced echelon form:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

3) Consider the vector space V of degree 2 polynomials with real coefficients:

$$V := \{a_0 + a_1 t + a_2 t^2 \mid a_0, a_1, a_2 \in \mathbb{R}\}.$$

(1) Write down two different bases of V. You must verify that what you produce are bases!

(2) Let $\mathcal{B} := \{1, t, t^2\}$ be an ordered basis of V. Define a map

$$L : V \to V$$
$$a_0 + a_1 t + a_2 t^2 \mapsto a_0 + a_1 t^2$$

Prove that L is a linear map. Write down the matrix representation of L with respect to the basis $\mathcal{B}$.

Date:

4) Suppose $L : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear map such that

$$L\left((1,0)\right) = (2,0)$$

and

$$L\left((0,1)\right) = (1,1).$$

Write down a formula for $L\left((x,y)\right)$ for any $(x,y) \in \mathbb{R}^2$.