PRACTICE EXAM TWO

Due on Friday, November 1

1) Compute the determinant and find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix}.$$

2) Consider the matrices,

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}.$$

Prove or disprove the following statement: A is conjugate to B.

3) Let A, B and C be 3×3 matrices with real coefficients and assume

$$\mathrm{Det}(A) = 30, \ \mathrm{Det}(B) = 2, \ \mathrm{Det}(C) = \frac{1}{2}.$$

(1) Compute $\mathrm{Det}(ABC)$.
(2) Let $\mathcal{C} \subset \mathbb{R}^3$ be the unit cube. Consider the regions enclosed by $A(\mathcal{C}), B(\mathcal{C}), C(\mathcal{C}) \subset \mathbb{R}^3$. Among them, which has the smallest volume?
(3) Suppose that A is diagonalizable with eigenvalues λ_1 and λ_2. Prove $\lambda_1 \lambda_2 = 30$.

4) Consider the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

(1) Find all the real eigenvalues and eigenvectors of A.
(2) Diagonalize A.
(3) Compute A^5 explicitly.

Date: