Math 180 Calculus 1

Exam 1 - July 11, 2014

Name:_____

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1: Let $f(x) = \dfrac{1}{x}$.

Find $f'(x)$ using **the definition of the derivative only.**

2: Let $f(t) = 5e^t - 2t^3 + \sqrt{t}$.

Find $f'(t), f''(t)$ and $f^{(3)}(t)$.

3: Compute the following limits, or explain why they do not exist.

a: $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x-1}$ **b:** $\lim\limits_{h\to 0} \dfrac{(h+3)^2-9}{h}$ **c:** $\lim\limits_{\theta\to 0} \dfrac{|\theta|}{\sin\theta}$

d: $\lim\limits_{t\to\infty} \dfrac{2t^2-100t+9}{t^3-8}$ **e:** $\lim\limits_{x\to-\infty} \dfrac{x^5+6x^4-11x+3}{16-x^2}$

4: Consider $g(x) = \dfrac{x^2+4x-12}{x^2-4}$.

a: Explain why $x = 2$ is **not** a vertical asymptote for $g(x)$.

b: Explain why $x = -2$ **is** a vertical asymptote for $g(x)$.

c: What are the horizontal asymptote(s) of $g(x)$?

d: Sketch a graph of $g(x)$.

5: The radius of a circle is shrinking at a constant rate of 2 cm/sec.

a: Find the rate at which the **circumference** of the circle is changing. Explain why the answer is negative.

b: Find the rate at which the **area** of the circle is changing when the radius is equal to 8 cm.

6: Use the squeeze theorem to show that $\lim_{x\to 0} x^2 \ln\left(\cos\left(\frac{1}{x}\right)+2\right) = 0$

7: Show that the equation $x^2 - e^{\cos(x)} = 0$ has a solution with $x \in [0, \pi]$.

Can you find a smaller interval that contains the solution?

8: A person located at point A on the edge of a lake in the shape of a circle of radius 1 mile wishes to reach the point C on the shore diametrically opposite. The person can row a boat at 3 mil/h and can jog at 6 mi/h. At what angle x with the diameter should the person row in order to reach C in the shortest possible time? (Hint: $\overline{AB}$ has length $2\cos x$. The length of arc $s = r2x = 2x$ since $r = 1$).