

QUIZ 1 SOLUTION

ALEXANDER J STATHIS

1. Solve the following system of equations:

$$\begin{aligned}x_1 - x_2 + 3x_3 + 2x_4 &= 1; \\ -x_1 - x_2 - 2x_3 + x_4 &= -2; \\ 2x_1 - 2x_2 + 7x_3 + 7x_4 &= 1.\end{aligned}$$

Solution. We begin by realizing the set of equations as the augmented matrix

$$\left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ -1 & 1 & -2 & 1 & -2 \\ 2 & -2 & 7 & 7 & 1 \end{array} \right),$$

and noticing that we have three equations in four unknowns, so it's very likely a solution exists unless something truly embarrassing happens.

We may row reduce the matrix without affecting the solutions to the equations, and indeed, the easiest method to solve such a system is to reduce the matrix to reduced row echelon form. We can immediately clear out the first and second columns of the bottom two rows by adding the first row to the middle row and subtracting twice the top row from the bottom one, and we obtain the matrix

$$\left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 3 & -1 \end{array} \right).$$

Subtracting the middle row from the bottom row completely clears it, and leaves us with the matrix

$$\left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right),$$

from which we can obtain

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & -7 & 4 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

in reduced echelon form by subtracting three times the middle row from the top row.

The fact that we have two pivots, one in each column corresponding to x_1 and x_3 , tells us that we have two free variables x_2 and x_4 , and that any set of solutions to our equations must satisfy both

$$x_1 - x_2 - 7x_4 = 4 \text{ and } x_3 + 3x_4 = -1.$$

The set of solutions is the set of vectors in \mathbb{R}^4 given by

$$\left\{ \begin{pmatrix} x_2 + 4x_4 + 4 \\ x_2 \\ -3x_4 - 1 \\ x_4 \end{pmatrix} \right\}$$

where x_2 and x_4 are free elements of \mathbb{R} .

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