

QUIZ 10 SOLUTIONS

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1. Find the best possible straight line to fit the data points

$$\{(0, 1), (1, 2), (2, 4) \text{ and } (3, 7)\}.$$

Solution by Prof K. We must solve the inconsistent linear system

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = (1 \quad 2 \quad 4 \quad 7)$$

in the least squares sense. To do this, we consider the equation

$$A^T A \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = A^T b$$

where A is taken to be the 4×2 matrix on the left hand side. The resulting system is represented by the augmented matrix

$$\left(\begin{array}{cc|c} 4 & 6 & 14 \\ 6 & 14 & 31 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 2 \end{array} \right),$$

where the row reduction is completed by halving the first row, subtracting thrice the first row from the second, subtracting three fifths the second row from the first, and then halving the first row again. The best fit line is given by the equation

$$y = 2x + \frac{1}{2}.$$

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