QUIZ 11 SOLUTIONS

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1. Use the Gram-Schmidt orthogonalization process to find an orthonormal basis for

$$S = \text{Span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ -4 \\ 0 \end{pmatrix} \right\}.$$

Solution. Because there are two vectors and neither is in the span of the other, we will need two basis vectors. Our first basis vector is

$$\mathbf{e}_1 = \frac{v_1}{\|v_1\|} = \frac{v_1}{\sqrt{1+4+4+16}} = \begin{pmatrix} 1/5 \\ 2/5 \\ 2/5 \\ 4/5 \end{pmatrix}.$$

Now we project our second vector onto our first, and subtract this projected vector from the second vector and normalize the result to obtain our second basis. The projection is

$$\text{proj}_{v_1}(v_2) = \frac{v_2 \cdot v_1}{\|v_1\|^2} v_1 = \frac{-2+0+-8+0}{25} v_1 = -\frac{2}{5} v_1 = \begin{pmatrix} -2/5 \\ -4/5 \\ -4/5 \\ -8/5 \end{pmatrix},$$

and the difference of v_2 and the projection is

$$u_2 = v_2 - \text{proj}_{v_1}(v_2) = \begin{pmatrix} -2 \\ 0 \\ -4 \\ 0 \end{pmatrix} - \begin{pmatrix} -2/5 \\ -4/5 \\ -4/5 \\ -8/5 \end{pmatrix} = \begin{pmatrix} -8/5 \\ 4/5 \\ -16/5 \\ 8/5 \end{pmatrix}.$$

Our second basis vector is

$$\mathbf{e}_2 = \frac{u_2}{\|u_2\|} = \frac{u_2}{\sqrt{\frac{64}{25} + \frac{16}{25} + \frac{(16)^2}{25} + \frac{64}{25}}} = \frac{u_2}{4} = \begin{pmatrix} -2/5 \\ 1/5 \\ -4/5 \\ 2/5 \end{pmatrix},$$

and summing up, our basis is

$$\left\{ \begin{pmatrix} 1/5 \\ 2/5 \\ 2/5 \\ 4/5 \end{pmatrix}, \begin{pmatrix} -2/5 \\ 1/5 \\ -4/5 \\ 2/5 \end{pmatrix} \right\}.$$

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