QUIZ 12 SOLUTIONS

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1. Let $A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$. Find the eigenvectors and eigenspaces of A. Determine if A is diagonalizable or defective.

Solution. We first need to find the characteristic polynomial of A. To do this, we consider the equation

$$0 = \det(A - \lambda I) = \det \begin{pmatrix} 3-\lambda & 1 \\ 2 & 4-\lambda \end{pmatrix} = (3-\lambda)(4-\lambda) - 2 = \lambda^2 - 7\lambda + 10 = (\lambda - 2)(\lambda - 5)$$

for arbitrary $\lambda \in \mathbb{R}$. Our eigenvalues are 2 and 5, accordingly.

To determine the eigenvectors for each eigenvalue, we just solve the system explicitly with each eigenvalue. In the case that $\lambda = 2$, we need to find the null space of

$$A - 2I = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}.$$

But this is just the line through the origin given by the equation $x = -y$, and has as a basis the vector $e_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Proceeding analogously for $\lambda = 5$, we find the vector $e_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

It follows that A diagonalizes to $\begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$ in the *ordered* basis $\{e_1, e_2\}$.

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