QUIZ 3 SOLUTION

ALEXANDER J STATHIS

1. Let A be the matrix

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 2 & -3 \end{pmatrix}.$$

Find the LU decomposition of A.

Solution. We begin by row reducing the matrix to an upper triangular matrix. We must be careful to keep track of the reductions we make along the way. First, add the top row to the bottom two rows to achieve the matrix

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & -2 \end{pmatrix},$$

and then subtract the twice the second row from the third row to get

$$U = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -6 \end{pmatrix}.$$

We performed three row operations corresponding to three elementary matrices E_1, E_2, and E_3, i.e.,

$$E_3E_2E_1A = U.$$

The matrices are

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix},\ \text{and } E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

with inverses

$$E_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix},\ \text{and } E_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}.$$

From above, it follows that

$$A = E_1^{-1}E_2^{-1}E_3^{-1}U = LU,$$

so that

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 2 & 1 \end{pmatrix}.$$

Notice that L is a lower triangular matrix.

2. Let B be the matrix

$$\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}.$$

Find B^{-1}.

Solution. Before we find B^{-1}, we note that $\det B = -1$, so that B is a nonsingular matrix. Now augment B on the right with an identity matrix, and perform row operations until B is the identity. The augmented matrix is

$$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array} \right).$$

Performing the program above, we arrive at the matrix

$$\left(\begin{array}{cc|cc} 1 & 0 & -5 & 3 \\ 0 & 1 & 2 & -1 \end{array} \right),$$

so that

$$B^{-1} = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}.$$

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