QUIZ 4 SOLUTION

ALEXANDER J STATHIS

1. Show that

$$S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : 2x + 3y = 0 \right\}$$

is a subspace of $\mathbb{R}^2$.

Solution. We must check that S is closed under sums and scalar products. If $c \in \mathbb{R}$ and $\begin{pmatrix} x \\ y \end{pmatrix} \in S$, then

$$c \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cx \\ cy \end{pmatrix}$$

and

$$2(cx) + 3(cy) = c(2x + 3y).$$

But $\begin{pmatrix} x \\ y \end{pmatrix} \in S$, so $2x + 3y = 0$, and therefore $\begin{pmatrix} cx \\ cy \end{pmatrix} \in S$ and S is closed under scalar products.

Now assume that $\begin{pmatrix} x \\ y \end{pmatrix}$ and $\begin{pmatrix} a \\ b \end{pmatrix}$ are elements of S so that $2a + 3b = 2x + 3y = 0$. Then

$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x + a \\ y + b \end{pmatrix}$$

and

$$2(x + a) + 3(y + b) = (2x + 3y) + (2a + 3b) = 0,$$

so S is closed under sums. Hence S is a subspace of $\mathbb{R}^2$.

2. Show that

$$T = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x = y \text{ or } x = z \right\}$$

is not a subspace of $\mathbb{R}^3$.

Solution. Observe that $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ are elements of T, but

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

is not an element of T. Therefore T is not closed under sums, and is not a subspace of $\mathbb{R}^3$.

Dept. of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, Chicago, IL 60607
E-mail address: astath2@uic.edu