

QUIZ 5 SOLUTION

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1. Let

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \text{ and } v_3 = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}.$$

1 Does $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ belong to $\text{Span}\{v_1, v_2, v_3\}$?

2 Do $v_1, v_2,$ and v_3 span \mathbb{R}^3 ?

Solution. First, we notice that an answer in the negative to question one suffices to show that the vectors do not span \mathbb{R}^3 since $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \in \mathbb{R}^3$.

To see if $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is in the span of these vectors, we need put them in the matrix

$$\begin{pmatrix} 1 & 3 & 2 & 0 \\ 2 & 4 & 2 & 0 \\ 1 & 5 & 4 & 1 \end{pmatrix}$$

and row reduce. If this matrix is inconsistent, then the vector lies outside the span of $v_1, v_2,$ and v_3 .

After row reducing, we arrive at the matrix

$$\begin{pmatrix} 1 & 3 & 2 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

which is inconsistent. Hence, $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is not in the span, and they do not span \mathbb{R}^3 .

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