

QUIZ 6 SOLUTION

ALEXANDER J STATHIS

1. Decide if $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \end{pmatrix} \right\}$ is a basis of \mathbb{R}^2 .

Solution. It suffices to determine whether the vectors are linearly independent or not, so we put them in the matrix

$$\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$$

and calculate the determinant. The determinant is $7 - 6 = 1$, so the matrix is invertible and the vectors are linearly independent. Therefore, they are a basis of \mathbb{R}^2 .

2. Give an example of a linearly dependent set in \mathbb{R}^3 .

Solution. There are plenty of examples of these. The set containing the zero vector, any set containing four vectors, or the set containing a vector and any scalar multiple of that vector all suffice.

3. Give an example of a linearly independent set in P_2 , the space of polynomials in a single variable with real coefficients of degree less than two.

Solution. Again, there are plenty of examples to choose from here. We could give the set containing the standard basis vectors, $\{1, x\}$, or we could give any set containing just a single nonzero vector.

DEPT. OF MATHEMATICS, STATISTICS, AND COMPUTER SCIENCE, UNIVERSITY OF ILLINOIS AT CHICAGO, CHICAGO, IL 60607
E-mail address: astath2@uic.edu