QUIZ 7 SOLUTION

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1. Consider the set

$$F = \left\{ \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ 7 \end{pmatrix} \right\}$$

as a basis of $\mathbb{R}^2$.

a If

$$u = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

in the standard basis of $\mathbb{R}^2$, find $[u]_F$.

b If

$$[v]_F = \begin{pmatrix} 2 \\ -1 \end{pmatrix},$$

find v in the standard basis of $\mathbb{R}^2$.

Solution.

a $F[v]_F = v$, so that $[v]_F = F^{-1}v$ and

$$F^{-1} = \begin{pmatrix} -7 & 5 \\ 3 & -2 \end{pmatrix}.$$

So

$$F^{-1}v = \begin{pmatrix} -7 & 5 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 27 \\ -11 \end{pmatrix}.$$

b We know that $F[v]_F = v$, so we just perform the multiplication

$$\begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = 2\begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}.$$

2. Let

$$F = \left\{ \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right\} \text{ and } G = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 9 \end{pmatrix} \right\}$$

be bases for $\mathbb{R}^2$. Compute the change of basis matrix from G to F.

Solution. We know that

$$F[v]_F = v = G[v]_G,$$

so that

$$G^{-1}F[v]_F = [v]_G$$

and $G^{-1}F$ is the change of basis matrix. Hence

$$G^{-1}F = \begin{pmatrix} 9 & -4 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 5 & 7 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 23 & 47 \\ -7 & -10 \end{pmatrix}.$$

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