

## QUIZ 8 SOLUTION

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1. Let  $L$  be the linear map from  $\mathbb{R}^3$  to  $\mathbb{R}^2$  given by

$$L \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x - 6y + 4z \\ -x + 3y - 2z \end{pmatrix}.$$

Find the standard matrix representation of  $L$ .

**Solution.** The matrix representation of  $L$  is

$$\begin{pmatrix} 2 & -6 & 4 \\ -1 & 3 & -2 \end{pmatrix}.$$

2. Find the kernel of  $L$ .

**Solution.** To find the kernel, we first row reduce

$$\begin{pmatrix} 2 & -6 & 4 \\ -1 & 3 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & 2 \\ 0 & 0 & 0 \end{pmatrix},$$

and conclude that the image of an element

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

is zero if  $x - 3y + 2z = 0$ , so the kernel is the plane in  $\mathbb{R}^3$  defined by this equation.

3. Find the range of  $L$ .

**Solution.** The dimension of the kernel is two, so the dimension of the image is one. More explicitly, it's pretty obvious by inspection that the columns of  $L$  are linearly dependent (they are all in the span of the first), so the image is  $\text{span} \left\{ \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\}$ .

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