QUIZ 9 SOLUTIONS

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Let $S = \text{span}\left\{\begin{pmatrix}1\\2\\0\\3\end{pmatrix}, \begin{pmatrix}0\\0\\3\\4\end{pmatrix}\right\}$, and let $T = \text{span}\left\{\begin{pmatrix}1\\-1\\0\\0\end{pmatrix}\right\}$.

1. Is $S \perp T$?

Solution. S is perpendicular to T if all the basis vectors of S are perpendicular to the basis vector of T since matrix multiplication is linear. But

$$s_1^T \cdot t = \begin{pmatrix}1 & 2 & 0 & 3\end{pmatrix} \cdot \begin{pmatrix}1\\-1\\0\\0\end{pmatrix} = -1$$

is nonzero, so they are not perpendicular.

2. Find $S^\perp$.

Solution. The null space of a matrix is the orthogonal complement of its row space (the set of all vectors whose dot product with the rows is zero). Therefore, we take the basis vectors of S and make them the rows in a matrix, then find the null space.

The resulting matrix is

$$\begin{pmatrix}1 & 2 & 0 & 3\\0 & 0 & 3 & 4\end{pmatrix}.$$

The null space is the following plane in $\mathbb{R}^4$

$$\left\{\begin{pmatrix}x\\y\\z\\w\end{pmatrix} \in \mathbb{R}^4 : x + 2y + 3w = z + \frac{4}{3}w = 0\right\},$$

or is given by the basis

$$\left\{\begin{pmatrix}-2\\1\\0\\0\end{pmatrix}, \begin{pmatrix}-3\\0\\-\frac{4}{3}\\1\end{pmatrix}\right\}.$$

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