

October 12

- (1) Find and classify critical points of the following:
 - (a) $f(x, y) = 9 - 2x + 4y - x^2 - 4y^2$
 - (b) $g(x, y) = e^x \cos y$
- (2) Find the volume of the largest rectangular box with edges parallel to the coordinate axes which can be inscribed in the ellipsoid

$$9x^2 + 36y^2 + 4z^2 = 36.$$

Solution: It suffices to maximize the volume of the eighth of the box which lies in the first octant. Any inscribed box is such that in the first octant, its dimensions are x by y by $z = \sqrt{9 - 9y^2 - (9/4)x^2}$, so the volume of the whole box is $V(x, y, z) = 8xy\sqrt{9 - 9y^2 - (9/4)x^2}$. It is easier to maximize the square of this, because you won't have to deal with the square root then. From here, it's a routine problem. In the end, you should get $\frac{16}{\sqrt{3}}$.

- (3) Quiz 6: Find the points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ such that the tangent plane is parallel to the plane $3x - y + 3z = 1$.

We've talked about this sort of problem quite a bit. If you didn't do the quiz perfectly, you should do it again. The two points, so you can check your work, are

$$\left(\frac{3\sqrt{2}}{5}, \frac{-\sqrt{2}}{10}, \frac{\sqrt{2}}{5}\right)$$

and

$$\left(\frac{-3\sqrt{2}}{5}, \frac{\sqrt{2}}{10}, \frac{-\sqrt{2}}{5}\right).$$