Group work problems for 10/8

- (1:00 class only) If $z = f(x, y)$, $x = x(s, t)$, and $y = y(s, t)$, write out the chain rule for $\frac{\partial z}{\partial t}$ and for $\frac{\partial z}{\partial s}$.

  solution is routine

- Find $\frac{\partial M}{\partial u}$ and $\frac{\partial M}{\partial v}$ where $M = xe^{y-z^2}$ and $x = 2uv$, $y = u - v$ and $z = u + v$.

  solution is again routine

- Show that any function of the form $z = f(x + at) + g(x - at)$ is a solution to

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2},$$

the wave equation.

Solution: The functions f and g are both functions of one variable, so let's make them look like it by substituting $u = x+at$ and $v = x - at$, giving

$$z = f(u) + g(v).$$

So to find $\frac{\partial z}{\partial t}$, we use the chain rule:

$$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial t}.$$

Note here that $\frac{\partial f}{\partial v} = \frac{\partial g}{\partial u} = 0$.

Then, using the fact that

$$\frac{\partial u}{\partial t} = a$$

and

$$\frac{\partial v}{\partial t} = -a,$$

we have

$$\frac{\partial z}{\partial t} = a\frac{\partial f}{\partial u} - a\frac{\partial g}{\partial v}.$$

To take $\frac{\partial}{\partial t}$ of this function, viewing $\frac{\partial f}{\partial u}$ as a function of u and $\frac{\partial g}{\partial v}$ as a function of v, we have using the chain rule carefully again,

$$\frac{\partial}{\partial t}\frac{\partial z}{\partial t} = a\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\right)\frac{\partial u}{\partial t} - a\frac{\partial}{\partial v}\left(\frac{\partial g}{\partial v}\right)\frac{\partial v}{\partial t}.$$

Therefore

$$\frac{\partial^2 z}{\partial t^2} = a^2\frac{\partial^2 f}{\partial u^2} + a^2\frac{\partial^2 g}{\partial v^2}.$$

Now, calculate $\frac{\partial^2 z}{\partial x^2}$ similiarly and you are done.