

Quiz 11

Due Wednesday, November 28 at the beginning of class.

The rules:

- Each question is worth ten points. The quiz will only be worth ten points total, so if you score more than ten points, it will count as bonus credit.
- You may work with other students, your uncle the engineer, your mom the mathematician, or anyone else that you know. What you turn in should be in your own words and you must write the names of the people you collaborated with under your name.
- You may use outside references if necessary. You must cite whatever references you use. Do not plagiarize. If you use a reference, you need to make it clear that you used the reference to better your understanding so that you could write up the solution in your own words.

- (1) Find the volume of the intersection of the three solid cylinders:

$$\begin{aligned}x^2 + y^2 &\leq 1, \\x^2 + z^2 &\leq 1, \text{ and} \\y^2 + z^2 &\leq 1.\end{aligned}$$

- (2) This problem concerns a subset E of the unit square $[0, 1] \times [0, 1]$ which is defined as follows:

- Subdivide $[0, 1] \times [0, 1]$ into nine equal squares. Define E_1 to be the middle square; that is $E_1 = [1/3, 2/3] \times [1/3, 2/3]$.
- Now subdivide the remaining squares into nine squares each. Define E_2 to be the union of the middle squares.
- Construct E_3, E_4 , and so on in the same fashion.
- Define E to be the infinite union of all of these:

$$E = E_1 \cup E_2 \cup E_3 \cup \dots$$

(See the illustrations on the next page.)

(a) What is the area of E ?

(b) Calculate

$$\iint_E f(x, y, z) dA,$$

where

$$f(x, y, z) = \begin{cases} \frac{1}{3^n} & \text{if } (x, y, z) \text{ is in } E_n \\ 0 & \text{otherwise} \end{cases}.$$

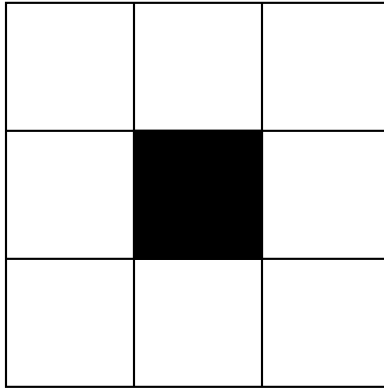


FIGURE 1. The black square is E_1

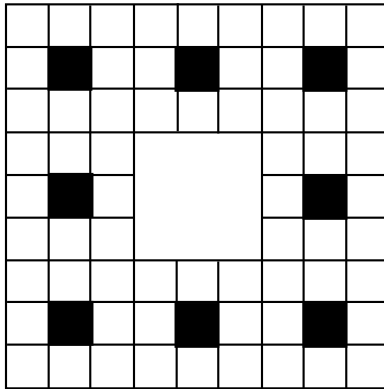


FIGURE 2. The union of the black squares is E_2

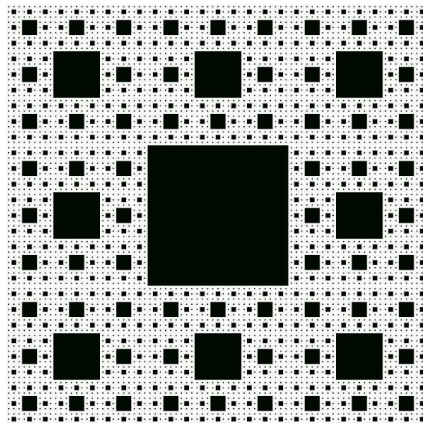


FIGURE 3. Here you can see a few more stages in the construction of E