

Another sample final

No guarantee of completeness. There will likely be types of problems on the final which do not appear here! Work out these problems before your discussion section.

- (1) Consider two interconnected tanks, A and B, each holding 32 L of a brine solution. Pure water flows into tank A at a rate of 8 L/min and brine solution flows out of tank B at the same rate. Brine solution is pumped from tank A to tank B at a rate of 16 L/min and from tank B to tank A at a rate of 8 L/min. If initially, tank A contains 4 kg of salt and tank B contains 8 kg of salt, determine the mass of salt in each tank for all time $t > 0$.

- (2) Find a general solution to

$$y'' + 2y' + 4y = e^{2t}$$

using the method of undetermined coefficients.

- (3) Solve the initial value problem using the Laplace transform:

$$y'' + 6y' + 9 = \delta(t - 3), \quad y(0) = 1, \quad y'(0) = 0.$$

- (4) Solve the integral equation for $y(t)$:

$$y(t) + \int_0^t (t - v)y(v) dv = t^2.$$

- (5) For what values of K does the problem have non-trivial solutions?

$$y'' + Ky = 0, \quad 0 < x < 2\pi$$

$$y'(0) = 0, \quad y(2\pi) = 0$$

- (6) Find the Fourier series for the following function:

$$f(x) = \begin{cases} 1 & \text{if } -\pi \leq x < 0 \\ -1 & \text{if } 0 \leq x \leq \pi \end{cases}$$

- (7) Compute the Fourier sine series for

$$f(x) = x, \quad 0 < x < 2\pi.$$

- (8) Solve the following heat equation for $u(x, t)$:

$$\frac{\partial u}{\partial t} = 3 \frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad 0 \leq x \leq 7$$

$$u(0, t) = u(7, t) = 0, \quad t > 0$$

$$u(x, 0) = \sin\left(\frac{\pi x}{7}\right) + 4 \sin\left(\frac{6\pi x}{7}\right) + 9 \sin(\pi x)$$